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博 士 論 文

基於矩陣李群的導航系統：可觀性分析與效能驗證
Observability Analysis and Performance Evaluation for Navigation Systems on Matrix Lie Groups

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摘要

論文題目：基於矩陣李群的導航系統：可觀性分析與效能驗證

研 究 生：曹書華

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本論文探討了如何利用矩陣李群 (matrix Lie groups) 來表達系統狀態以及對應的李代數 (Lie algebras) 用來處理狀態擾動量 (或不確定性)。我們將此數學工具應用於三個實際的例子來驗證其有效性。在第一個例子中，我們分析不同擾動模型對基於卡爾曼濾波器 (extended Kalman filter, EKF) 視覺–慣性里程計 (visual–inertial odometry, VIO) 的效能影響。從可觀性及精度分析的角度來看，實驗結果指出採用右不變 (right-invariant) 模型的 VIO 性能最佳。在第二個例子中，我們將基於因子圖最佳化 (factor graph optimization) VIO 框架中的預積分 (preintegration) 技術與此數學工具整合。在一致性 (consistency) 測試與定位性能評估中，整合右不變模型的 IMU 預積分 VIO 優於傳統模型的 IMU 預積分 VIO。在第三個例子中，我們透過全球導航衛星系統 (global navigation satellite system, GNSS) 中的偽距 (pseudorange) 與都卜勒位移 (Doppler shift) 觀測量進一步提升 VIO 的效能。最後，我們以低成本智慧型手機為平台，用來驗證本論文所提出的估測器效能。為了取得此手機中的感測器參數，我們實現了一基於 RTS (Rauch–Tung–Striebel) 平滑器的校正演算法。我們利用基於手機感測器的資料集來驗證估測器是否符合理論分析及評估定位精度。

關 鍵 字：狀態估計、可觀性分析

Abstract

Title: Observability Analysis and Performance Evaluation for Navigation Systems on Matrix Lie Groups

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This dissertation explores the use of expressing system states on matrix Lie groups with associated uncertainties on Lie algebras. In particular, we demonstrate the effectiveness of this mathematical tool by applying it to three realistic applications with observability analyses. In the first application, we proposed an extended Kalman filter (EKF)-based visual–inertial odometry (VIO) with different strategies of uncertainty perturbations. Experimental results demonstrate that the right-invariant (RI)-based VIO achieved the best performance. This can be attributed to the system's ability to remain consistent even when evaluating Jacobians using the most recent state estimates. In the second application, we extended a common technique in the factor graph optimization (FGO)-based VIO frameworks, namely inertial measurement unit (IMU) preintegration using the RI state error. Consistency tests and localization performance evaluations demonstrate the effectiveness of integrating the proposed IMU preintegration factor into FGO-based VIO frameworks. In the third application, we improved the robustness of VIO with aiding measurements from global navigation satellite systems (GNSS). We present the improved performance due to the complementary characteristics obtained from pseudorange and Doppler shift observations, respectively. Finally, we presented real-world data sets collected from a low-cost device, i.e., smartphone. We first implement an Rauch–Tung–Striebel (RTS) marker-based algorithm on matrix Lie groups to calibrate the necessary parameters. Additionally, we verify the theoretical analyses and localization performance for the implemented estimators using experimental results from self-collected data sets.

Keywords: State estimation, observability analysis

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x	Real scalars.
$\mathbf{x}$	Real column vectors.
$\mathbf{X}$	Real matrices.
$\dot{(\cdot)}$	Continuous time derivatives.
$\check{(\cdot)}$	Prior values.
$\hat{(\cdot)}$	(Posterior) estimated values.
$\tilde{(\cdot)}$	Error states.
$\{\cdot\}$	Coordinate frames.
${}^{B}(\cdot)_{A}$ or ${}^{B}_{A}(\cdot)$	The quantity of A expressed in coordinate frame $\{B\}$.
$\mathbf{0}_{m\times n}$	$m \times n$ zero matrices.
$\mathbf{1}_{m\times n}$	$m \times n$ identity matrices.
$[\cdot]_{\times}$	A 3×3 skew-symmetric matrix associated with the cross product, i.e., $[\mathbf{a}]_{\times}\mathbf{b} = \mathbf{a} \times \mathbf{b}$.